Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Real Analysis

Subject Code: 5SC02REA1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 19/09/2022** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a) Define: Algebra of sets (02)

b) Define: F_σ - set (02)

c) If $A, B \subseteq R$ be such that $m^*(B) = 0$ then prove that $m^*(A \cup B) = m^*(A)$. (02)

d) What is the measure of [2,5)? (01)

Q-2 **Attempt all questions** **(14)**

a) Let $\mathcal{A}$ be an algebra on X and $\{A_i\} \in \mathcal{A}$ then there exist $\{B_i\} \in \mathcal{A}$ such that (07)

i) $\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} B_i$ and ii) $B_i \cap B_j = \phi$, for $i \neq j$.

b) Prove that outer measure of an interval is its length. (07)

OR

Q-2 **Attempt all questions** **(14)**

a) Prove that P is non-measurable set. Where P contains one element from each equivalence classes E_λ and $\cup E_\lambda = X = [0,1)$. (09)

b) Consider $X = R$ and $\mathcal{A} = \{A \in R / \text{either A or A}^c \text{ is countable}\}$ then show that $\mathcal{A}$ is a σ-Algebra on R. (05)

Q-3 **Attempt all questions** **(14)**

a) If $E_1, E_2, ..., E_n$ be a finite sequence of measurable sets and they are mutually disjoint then for any $A \subseteq R$, $m^*\left(A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$. (05)

b) Let ϕ and ψ are simple functions on E which are vanish outside of a set of finite measure then prove that $\int a\phi + b\psi = a\int \phi + b\int \psi$. (05)

c) Give an example of an algebra which is not an σ-Algebra of X and explain. (04)

OR

Q-3 **Attempt all questions** **(14)**

a) Let f be a bounded function on a measurable set E with $m(E) < \infty$. if $\inf_{\psi \geq f} \int_E \psi = \sup_{\phi \leq f} \int_E \phi$, for all simple functions ϕ and ψ on E then prove that f is a measurable functions on E. (05)

b) If $E_1, E_2, ..., E_n$ be a finite sequence of measurable sets and they are mutually disjoint then for any $A \subseteq R$, $m^*\left(A \cap \left(\bigcup_{i=1}^{n} E_i\right)\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$. (05)

c) Let E_1 and E_2 be two measurable subsets of R then prove that $m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$. (04)

SECTION – II

Q-4 **Attempt the Following questions** **(07)**

a) Explain monotone convergence theorem is false for decreasing function. (02)

b) State Fatou's lemma. (02)

c) State Egoroff's theorem. (02)

d) True/False:A real valued function f is said to be of bounded variation on [a,b] if total variation is finite. (01)

Q-5 **Attempt all questions** **(14)**

a) State and prove Bounded convergence theorem. (07)

b) State and prove Lebesgue dominated convergence theorem. (07)

OR

Q-5 **Attempt all questions** **(14)**

a) State and prove monotone convergence theorem. (07)

b) Define convergence in the sense of measure and also prove that there is a subsequence $\{f_{n_k}\}$ of $\{f_n\}$ pointwise converges to f a.e. on E. (04)

c) State Littlewoods's three principles. (03)

Q-6 **Attempt all questions** **(14)**

a) State and prove Fundamental theorem of integral calculus. (07)

b) If f isintegrable over E then $|f|$ is integrable over E and $\left|\int_E f\right| \le \int_E |f|$. (05)

c) Write Chebychev's inequality. (02)

OR

Q-6 **Attempt all Questions** **(14)**

a) F is absolutely continuous function on $[a,b]$ iff F is indefinite integral. (07)

b) Suppose $f, g \in BV[a,b]$ then prove the following: (07)

i) $fg \in BV[a,b]$ and ii) $\frac{f}{g} \in BV[a,b], where\ g \neq 0$.